



## 2011-2012 Puzzle Contests Solutions for Contest #2



### **Parents and Grandparents Puzzle Solutions:**

#### **1. FIND ANY SOLUTION**

*ARE THERE ANY* whole solutions for the equation  $3x^2 + 2 = y^2$ ? Find it if possible. (30 pts)

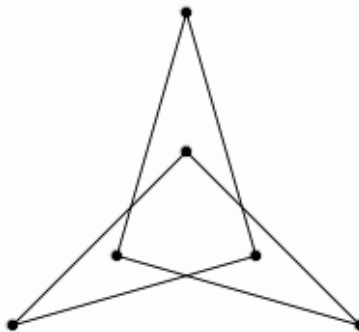
Answer: That equation has no whole solutions.

Proof: Assume it has a solution and the given identity is true for it. Then It follows from the equation that  $y^2$  is not divisible by 3. It means that  $y$  is not divisible by 3. Then note that  $y^2 = 3 \cdot k + 1$ . Hence for any  $x$   $3x^2 = 3 \cdot k - 1$ . That is impossible due to the different divisibility by 3 of both sides of the last identity

#### **2. EXTRAORDINARY BROKEN LINES**

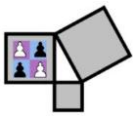
*IS IT POSSIBLE* to construct a) 6-link (15 pts) b) 7-link (15 pts) closed broken lines in which each link intersects exactly one of the others? Show if it is possible or explain why not.

Answer: a) it is possible. The possible shape of 6-link closed broken line is shown below:



b) it is impossible:

Proof: Assume it is possible to construct such 7-link closed broken line. Then intersecting links form the pairs. Hence the number of links has to be even. That contradicts to the condition of the problem.



### ***3. ELECTIONS IN THE CITY OF MATHEMATICS***

***11 PARTIES PARTICIPATE*** in the Elections for the 55-seat Government of the City of Mathematics. According to the Election Law each party must get more than 5% of the votes to be elected to the Government. The seats in the Government are allocated proportionally to the number of votes received by each elected party. After the Elections it was announced that each voter had voted for exactly one of the parties (no invalid ballots, no votes “against all” etc.). The Party For the Promotion of Math Puzzles won 25% of all votes. What is the greatest number of seats in the Government that the Party For the Promotion of Math Puzzles could possibly get? (***40 pts***)

Answer: 25 seats.

Solution: The Party For the Promotion of Math Puzzles could get the greatest number of seats if the number of parties which did not pass the 5%-barrier is the greatest possible, and each of them wins exactly 5% of the votes. If 9 parties do not 5% of votes for each, then 2 parties (The Party For the Promotion of Math Puzzles among them) could take 25% and 30% of votes. These results give 25 and 30 seats respectively. It was impossible for 10 parties to not pass the 5%-barrier because The Party For the Promotion of Math Puzzles could not get 25% of votes then ( $10 \times 5\% + 25\% < 100\%$ )